論文

Ply Cracking in Composite Laminates : Phenomenon and Modeling

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복합재 적층판의 섬유방향 균열에 관한 고찰 : 현상과 모델링

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ABSTRACT

Ply cracking in composite laminates occurs as a result of the low transverse tensile strength, and is the most common subcritical failure mode preceding ultimate failure. It is frequently observed in thermoset matrix composites and even in thermoplastic matrix composites. As a result, extensive research has been done on the prediction of first ply cracking as well as subsequent ply cracking. However, how one model differs from another is not always clear, adding confusion to the practicing engineers. In the present paper, therefore, we provide a critical review of different models that have been proposed, Specifically, we delineate assumptions regarding geometry, material property, fracture criterion, stress analysis and stiffness degradation.

초 록

복합재 적층판의 섬유방향 균열은 낮은 횡방향 강도에 기인하며, 최종 파단전에 발생하는 파괴 양상중의 하나이다. 이는 열경화성 수지 뿐만 아니라 열가소성 수지 복합재 적층판에서 조차도 종종 발견되며 하중의 증가에 따라 적층판의 강성저하 및 환경특성 저하를 초래하므로 구조설계시 고려하여야 할 중요한 인자중의 하나이다. 따라서 섬유방향 균열의 진전 및 그 영향에 대하여 상당히 광범한 연구가 진행되어 왔으며 특성 예측을 위하여 많은 모델들이 제시되어 왔다.

본 연구에서는 지금까지 제시된 많은 모델들의 특성과 차이점을 비교 분석하였으며, 특히 각 모델들의 기하학적 형상, 기계적 성질, 파단조건, 응력해석, 강성저하 등과 관련된 여러가지 가정들을 기술, 장단점을 비교하였다.

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1. Ply Cracking in Composite Laminates : Phenomenon

When a laminate is loaded, ply cracks parallel to the fibers form in the weakest ply long before the ultimate laminate failure(Fig. 1). These cracks are frequently observed in thermoset matrix composites and even in thermoplastic matrix composites. Since the matrix and interface are much weaker than the fiber, these cracks always propagate through the matrix and interface. Furthermore, the actual crack path is quite complex rather than straight and smooth as shown in Fig. 2. Growth of ply cracks is sooner or later prevented by the intersecting fibers in the adjacent plies, i.e., ply cracks are arrested at the ply interface. This crack arresting mechanism permits the applied load to increase without catastrophic propagation of ply cracks. As the load increases further, more ply cracks develop. The resulting crack density increases with the applied load in quasi-static tension, or with the applied cycles in fatigue. This ply crack multiplication continues until the main load-carrying fibers begin to break and delamination occurs between the cracked plies and the adjacent plies (Fig. 2). Soon thereafter final failure follows. Thus, in the absence of delamination, the density of ply cracks is the primary parameter representing the damage state.

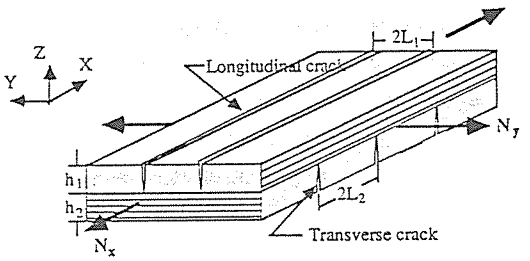

Fig. 1 Cross-ply laminate under biaxial loading.

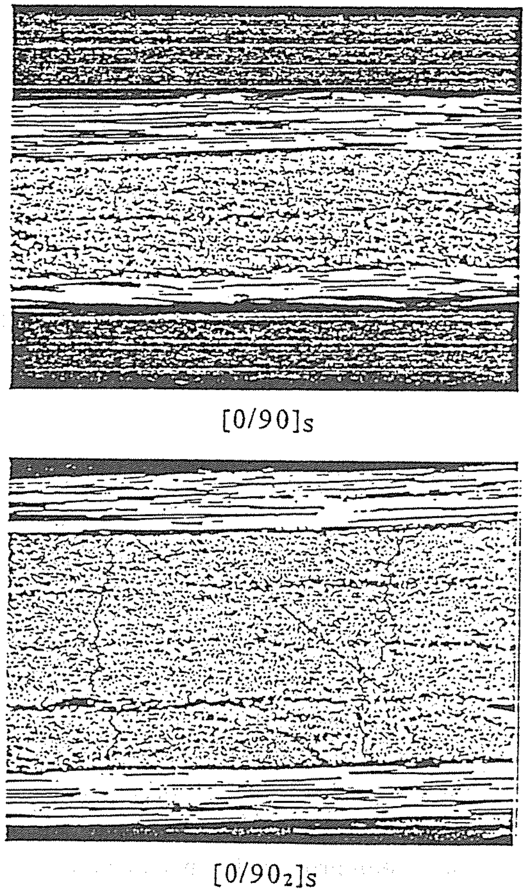

Fig. 2 Transverse ply cracks in cross-ply laminates.

Although these ply cracks do not lead immediately to catastrophic failure, their presence can be deleterious. They degrade mechanical properties, cause leakage in pressure vessels, and accelerate corrosion in wet or corrosive environments. Thus, it is important to understand ply cracking behavior in order to imporove the structural integrity of composite laminates.

2. Ply Cracking Under Static Loading : Modeling

Understanding of the ply cracking behavior leads

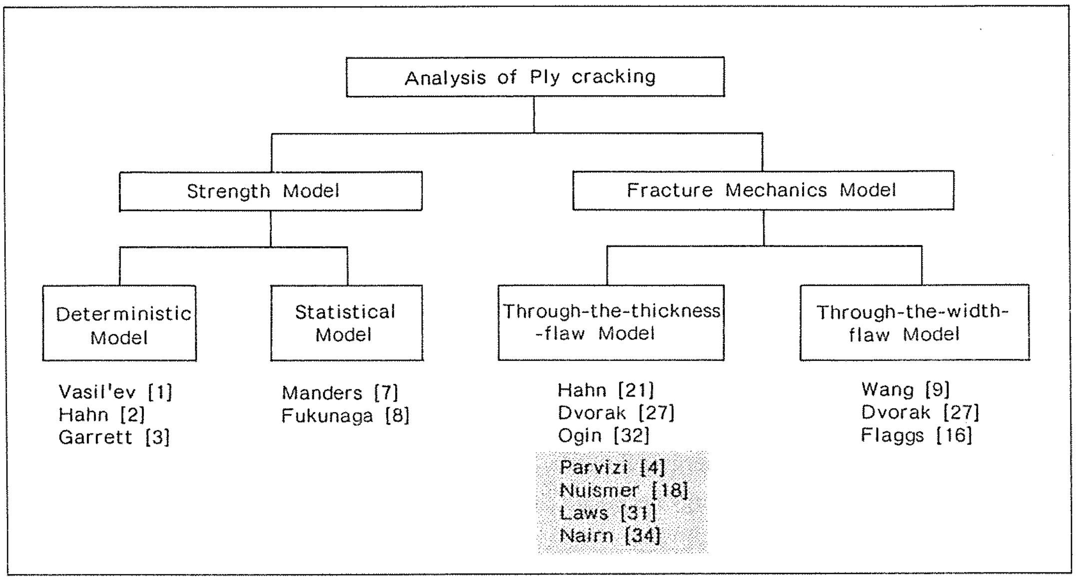

Fig. 3 Flow chart for the methods of analyzing transverse ply cracks.

to the following four questions : 1) When does a crack form in the transverse ply ? 2) How do the crack densities increase ? 3) What is the resulting degradation in porperties ? and 4) What is the pattern of ply cracking ?

Since the early 1970's, many models have been proposed to predict ply cracking. In general, the existing models may be grouped into two categories : strength models and fracture mechanics models(Fig. 3). The strength models assume that ply cracking is the result of the ply stress exceeding the ply strength[1-8]. On the other hand, the fracture mechanics models are based on the assumption that ply cracking is the result of the growth of inherent flaws, which is governed by the balance of energy principle[4, 9-35]. The former can be further divided into deterministic and statistical models. The latter can be divided into through-the-thickness and through-the-width flaw models. Some models were developed to predict progressive ply cracking as well as first ply cracking[1, 3, 7, 8, 9, 18, 21, 31, 34] whereas others were only for first ply cracking[4, 16, 27, 32].

The analyses involved in these models are by necessity based on several simplifying assumptions regarding geometry, material property, stress analysis, and fracture criterion. Specifically, the pertinent parameters are :

- Shape and size of crack
- Crack spacing
- Shape and size of inherent flaw
- Ply strength
- Energy release rate
- Fracture toughness
- Fracture criterion
- Method of stress analysis

In the following discussion, assumptions regarding these parameters are delineated to distinguish various models from one another.

2-1. First Ply Cracking

Fracture criterion :

In the strength models, the key material parameter is the ply strength. The ply strength was assumed deterministic and equal to the bulk transverse strength[1-3]. The new crack was assumed to occur when the stress in 90° plies is the same as its transverse tensile strength as shown in Eq. (2.1).

$$\sigma(90°)=X \quad \cdots\cdots (2.1)$$

where σ(90°) is the stress in 90° plies and X is transverse tensile strength.

In the fracture mechanics models, the key parameter is the fracture toughness. The new crack was assumed to occur when the energy released by the cracking is the same as the energy necessary to create new crack surfaces(Eq. 2.2).

$$G=G_c \quad \cdots\cdots (2.2)$$

where G and G_c is the energy release rate and fracture toughness, respectively.

As mentioned above, these models can be further divided into through-the-thickness and through-the-width flaw models. The way to calculate the energy release rate depends on the type of inherent flaw. For the through-the-width flaw, the energy release rate becomes a function of the crack size in the thickness direction. Thus, one needs to evaluate the energy release rate as a function of the flaw size. For the through-the-thickness flaw, the energy release rate can be calculated from either the required work of crack closure or the difference of the potential energy before and after the additional crack formation.

Vasil'ev et al.[1] and Garrett and Bailey[3] tried to explain the ply crack formation using deterministic strength criterion. The frist ply cracking strains were found to be independent of laminate stacking sequence since the stress in a 90° ply remained same as long as the same uniaxial strains were applied. However, experimental results[4, 16] showed a strong dependence of cracking strain on the adjacent ply stiffness and transverse ply thickness. The first ply cracking strain is known to increase as the thickness of the cracked ply decreases and the stiffness of the adjacent ply increases.

Fracture mechanics models based on the presence of inherent flaws have been found to be able to explain the dependence of the cracking strain on the stiffness of adjacent plies and the thickness of the cracked ply[4, 9-35]. Wang et al. [9-15] assumed that the formation of a crack was due to the growth of a through-the-width inherent flaw. The two parameter Weibull distribution was used to determine inherent flaw distribution and those parameters were adjusted from the experimental data. A finite element method was used to obtain the energy release rate. The authors[21-26] assumed that through-the-thickness flaws are the origin of ply cracks. It was shown that the energy release rate associated with the widthwise growth of a through-the-thickness flaw is equal to the work of crack closure per unit width of the fully developed flaw. The energy release rate is independent of the flaw length and hence the growth of the flaw is stable.

Dvorak and Laws[27-30] considered a flaw which is both partially through the thickness and partially through the width. The required stress analysis is three-dimensional, unlike the aforementioned two models. Two directions of the crack growth and the corresponding energy release rates were calculated, i.e., parallel to the fiber axis and parallel to the thickness direction of the laminate. However, the size and shape of the inherent flaw are hard to determine. In Fig. 3, the models in the shaded area did not specify and flaw growth mode ; however, these models were lumped into the through-the-thickness flaw models since the way to calculate energy release was the same as for the through-the-thickness flaw models.

It should be noted that all assumed inherent flaws are planar and normal to the laminate. Although the actual flaws may not be so smooth(Fig. 2), a simple and regular geometry had to be used for the flaws for ease of analyses. All models neglected delamination at the interface between the cracked ply and the neighboring plies.

Symmetric cross-ply laminates were object of most extensive study although laminates of $[\pm(\theta)_i/90_j]_s$ and $[90_i/\pm(\theta)_j]_s$ types were studied to some extent by the authors[24]. For analysis purposes, the angle plies were lumped together as an orthotropic sublaminate.

Method of Stress Analysis :

Once fracture criteria are defined, the next step is how to obtain either the stress in a 90° ply or the energy release rate for the new crack formation of a specific laminate, i.e., stress analysis. A number of methods have been used to obtain the stress field of a cracked laminate(Fig. 1). They can be categorized as follows :

1) Minimization of potential energy ; Vasil'ev et al.[1].

2) Parallel spring model ; Hahn and Tsai[2]

3) Shear-lag/approximate elasticity model ; Garrett, Bailey, and Parvizi[3-6], Fukunaga et al.[8], Flaggs[17], Nuismer and Tan[18-20], Hahn and Han[22-26], Laws and Dvorak[31], Highsmith and Reifsnider[39], Aboudi[49,50].

4) Self-consistent scheme ; Dvorak and Laws [27-30]

5) Minimization of complementary strain energy ; Nairn[34,35], Hashin[47,48]

6) Internal variable/damage mechanics model ; Talreja[40-42], Allen et al.[43-46].

These analyses can also be classified into two groups according to the methods which were used to formulate the constitutive relations of the cracked laminate. The first approach is to evaluate the stress distribution of a representative volume element containing a single crack, and to deduce the relationship between the applied load and laminate overall response. The stress distribution will satisfy the boundary conditions and equilibrium equations around the cracked area. Shear-lag/approximate elasticity and minimization of energies belong to this group. The second approach is to evaluate the averaged constitutive properties from the damaged representative volume element rather than the calculate stress distribution which satisfies boundary conditions and equilibrium equation. The self consistent scheme, continuum damage mechanics model, and parallel spring model are of this nature.

Each model has its own advantages and disadvantages. Some models are for general in-plane loading[20,26,40,43,47,49] while others are for uniaxial loading[31,39]. Some models are more versatile in the laminate stacking sequence[28,39,43] while other are not due to the mathematical complexity[31,35,47,49]. Some models have parameters that need to be adjusted experimentally or empirically[8,15,27,39,40].

As shown in the above categorization, extensive work has been done on the shear-lag/approximate elasticity approach to obtain the stress distribution in a cracked laminate. The typical procedure in this analysis is to set up the governing differential equation from 1) the assumed displacement field, 2) compatibility and equilibrium equations, 3) constitutive euqation. However, some models needed to adjust a parameter if the displacement discontinuity was allowed between sublaminates.

The other approach is the minimization of energy used by Vasil'ev et al.[1] and Hashin[47-48]. Assuming the stress in each sublaminate is constant along the thickness direction, they formulated the complementary energy[47] and potential energy functional[1]. The perturbed stress field was then obtained by minimization of the energy. The mathematical complexity which arises during the minimization procedure places a limit on the admissible stress field and geometries other than cross-ply laminates are difficult to analyze. It is noted that the governing equation of the variational approach is a 4th order differential so that the resulting stress field becomes hyperbolic and trigonometric functions. Conversely, the equation of the shear-lag/approximate elasticity model is 2nd order so that the field becomes a hyperbolic function. It is also noted that the perturbed stress σ_x in the sublaminate 2(90° plies) has a peak, which is larger than nominal stress. In finite element analysis, this stress approaches the nominal value asymptotically.

Dvorak and Laws[27-30] used a self-consistent scheme to obtain the elastic stiffness of the damaged lamina. The elastic moduli of a cracked ply were assumed to be equal to those of a similarly cracked infinite fibrous material. The stiffness of this infinite medium with a crack was calculated using a self-consistent calculation. Then, the overall laminate stiffness was calculated from classical laminated plate theory.

Talreja[40-42] and Allen et al.[43-46] have developed a model for predicting stiffness loss as a function of the damage state, which was represented by a set of internal state variables. Allen et al. defined an internal state variable as the average value of the crack opening displacement multiplied by the unit normal to the crack surface. Talreja defined a damage tensor in a similar way. One advantage of this approach is its applicability to different laminate stacking sequences. However, Talreja's approach requires determining certain phenomenological constants from experimental data on one laminate of known damage, which are then used for predicting stiffness changes for other laminates.

Ins summary of the various stress analyses, the key assumptions made for the analysis are listed in Table 1. One important aspect is the assumption on displacement or stress field. Some models assumed that the displacement or the stress of each sublaminate is constant along the thickness direction, which is obviousley far from the real field. This displacement field was evolved to assume a linear or 2nd order polynomial through the thickness direction. The second order polynomial displacement field, which was proposed by authors, was verified to be accurate by the finite element method[23]. Another aspect is the consideration of an out-of-plane stress term. Some models included these stresses during the analysis while others did not(Table 1).

Discussion :

In Fig. 4, the first ply cracking strain predicted from the five fracture mechanics models are compared with the experimental results for $[0_2/90_n]_s$ T300/934 laminate family. The material properties used in this analysis are in Ref.[15, 17]. Note that the ordinate of Fig. 4 is the in-situ transverse strain, which includes the mechanical strain as well as the thermal strain. The predictions from through-the-thickness flaw models agree well with experimental results when there are fewer than six 90° plies. But the deviation from the experimental results increases as the thickness of 90° plies increases. Flaggs[17] reduced this discrepancy by adjusting the critical flaw size. However, the 2-D shear-lag analysis which he employed was for the through-the-thickness flaw and the concept of the critical flaw size is for the through-the-width flaw.

The reason of increasing deviation in Fig. 4 may be that the mode of transverse ply cracking changes as the thickness of the transverse ply changes. When the 90° ply is thin, the assumption of the

Table 1. Comparison of the models for ply cracking in composite laminates

Questionnaire

1) How were displacements in the sublaminates assumed?
2) Must any variables be determined in addition to the basic engineering constants?
3) Was the model developed for general in-plane loading?
4) Does the model calculate out-of plane stresses?
5) Does the model predict stiffness?
6) Does the model predict crack initiation and/or multiplication under tensile loading? If yes, What criterion does it use?
7a) If the model predicts crack multiplication, does the model allow the gradual crack multiplication from N to N+1 crack density?
7b) Does the model use a deterministic value for strength or fracture toughness?
8) Does the model predict progressive ply cracking under combined loading?
9) Is the model based on crack growth mechanism?

	Stress analysis	Stiffness	Failure analysis
Vasil'ev et al. [1]	1) const.for sublaminates 1,2 2) no 3) tensile load only 4) yes	5) Young's modulus	6) both, strength criterion 7a) no, N, 2N, 4N, 8N...type. 7b) deterministic strength 8-9) no
Hajn & Tsai [2]	1) const.for sublaminates 1,2 2) no 3) tensile load only 4) no	5) Young's modulus	6-9) no
Garrett, Bailey & Parvizi [3-6]	1) const.for sublaminate 1 linear for sublaminate 2 2) no 3) tensile load only 4) no	5) Young's modulus	6) strength criterion for crack multiplication, energy 7a) no, N, 2N, 4N, 8N...type. 7b) deterministic strength 8-9) no
Wang & Crossman [9-15]	1) finite element analysis 2) statistical distribution of flaws 3) tensile load only 4) yes	5) no	6) both, energy balance 7a) yes, N, N+1, N+2, ... type 7b) stochastic simulation 8) no 9) yes, through-the-width flaw.
Highsmith Reifsnider [39]	1) const. for sublaminates 2) shear layer thickness 3) tensile load only 4) no	5) Young's modulus	6-9) no
Flaggs [17]	1) linear in sublaminates 1,2 2) no 3) general in-plane loading 4) yes	5) stiffness matrix	6) first ply cracking, energy balance 7) no 8) yes 9) yes, only for first ply cracking.
Nuismer & Tan [18-20]	1) const.for sublaminates 1,2 (used average values) 2) no 3) general in-plane loading 4) yes	5) Young's and shear moduli, Poisson's ratio.	6) both, energy balance 7a) no, N, 2N, 4N, 8N...type. 7b) deterministic toughness 8-9) no
Han & Hahn [21-26]	1) const. for sublaminate 1 2nd order polynomial for sublaminate 2 2) no 3) general in-plane loading 4) no	5) Young's and shear moduli, Poisson's ratio	6) both, energy balance 7a) yes, N, N+1, N+2, ... type 7b) statistical toughness distribution 8) yes 9) yes, through-the-thickness flaw
Manders et al. [7]	1-4) used Garret, Bailey & Parvizi model.	5) no	6) both, strength criterion 7a) no, N, 2N, 4N, 8N...type. 7b) statistical strength distribution 8-9) no
Fukunaga et al. [8]	1) const.for sublaminates 1,2 2) shear transfer layer thickness 3) tensile load only 4) no	5) no	6) both, strength criterion 7a) no, N, 2N, 4N, 8N...type. 7b) statistical strength distribution 8-9) no
Dvorak & Laws [27-30]	1) - 2) reduction coefficients 3) general in-plane loading 4) no	5) stiffness matrix	6) first ply cracking, energy balance 7-8) no 9) yes.through-the-thickness & through-the-width inherent flaws.

Laws & Dvorak [31]	1) const.for sublaminates 1,2 2) shear lag parameter 3) tensile load only 4) no	5) Young's modulus	6) both, energy balance 7a) no, N, 2N, 4N, 8N...type. 7b) probability density function. 8-9) no
Talreja [40-42]	1) - 2) material constant for damage (ki) 3) general in-plane loading 4) yes	5) Young's and shear moduli, Poisson's ratio	6-9) no
Allen et al. [43-46]	1) trigonometric function for sublaminate 2 2) no 3) tensile load only 4) yes	5) Young's modulus	6-9) no
Hashin [47-48]	1) const.for sublaminates 1,2 2) no 3) general in-plane loading 4) yes	5) Young's and shear moduli, Poisson's ratio	6-9) no
Ogin & Smith [32-33]	1-4) based on shear lag analysis.Detailed information was not avilable.	5) Young's modulus	6) first ply cracking, stress intensity factor 7-8) no 9) yes, through-the -thickness & through-the-width inherent flaw
Aboudi [49-50]	1) 2nd order polynomial for sublaminate 1,2 2) no 3) general in-plane loading 4) yes	5) stiffness matrix	6-9) no
Nairn [34-35]	1-4) used Hashin's model	5) no	6) both, energy balance 7a) no, N, 2N, 4N, 8N, ... type 7b) deterministic toughness 8-9) no

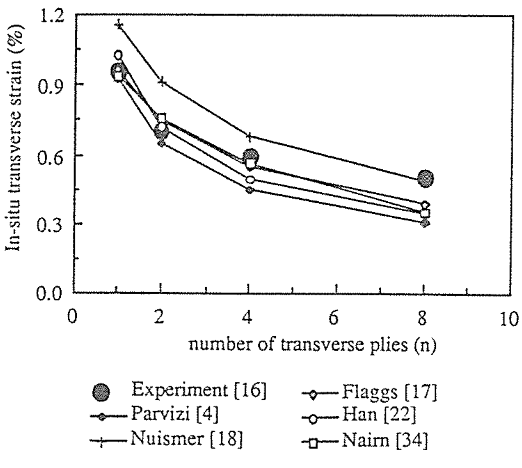

Fig. 4 Predictions of first ply cracking strains in T300/934 $[0_2/90n]s$ laminates.

inherent flaw being a through-the-thickness type is reasonable. However, as the 90° ply becomes thicker, the same assumption is not likely to hold true. Since the analytical models are based on the presence of a through-the-thickness flaw, their predictions are expected to become worse as the 90° ply thickness increases. For thicker 90° plies, one may use an inherent flaw which is neither through-the-thickness nor through-the-width, as was studied by Dvorak and Laws[27]. Nairn[34] showed better experimental correlation by adjusting toughness, transverse shear modulus, and stress free temperature. He used a different value from the G_{IC} for the critical value of crack formation. The value was determined from the best fit curve of first ply cracking test results and found to be $89J/m^2$ as opposed to $158J/m^2$ which was obtained from the DCB tests[15].

Both ply cracking and delamination appear to be governed by the same fracture toughness. Lee [36] simulated delamination and transverse ply cracking using the width tapered double cantilever beam and the double torsion specimen, respectively. He found the fracture toughness values obtained from the two tests to be comparable. Thus, he concluded the two failure modes had the same failure mechanisms. The fracture toughness from a 0-degree DCB test can be used to predict ply cracking because the crack growth direction is parallel to the fibers. It should be noted that Wang [14] used a fracture toughness for normal crack propagation obtained from a 90° DCB test since his through-the-width flaw propagated normal to the fibers.

2-2. Progressive Ply Cracking

In the strength models, the transverse strength was assumed either to be deterministic[1, 3] or to vary statistically[7, 8] for the prediction of progressive ply cracking. Manders et al. [7] and Fukunaga et al. [8] used statistical strength distributions in their strength models whereas Vasil' ev[1] and Garrett[3] used deterministic strengths. The transverse strength was assumed to obey a two-parameter Weibull distribution. It was assumed that a new crack occurred midway between any two adjacent cracks at 50% failure probability.

In the fracture mechanics models, the fracture toughness was assumed to be either deterministic and homogeneous[4, 9-20, 27-35] or inhomogeneous[22-26]. In the latter model which was proposed by the authors, the fracture toughness was called "resistance to ply cracking" which represents the inhomogeneous nature of fracture energy in the composite laminates. The way to calculate the energy release rate is also different from each model.

The through-the-width flaw model assumed that the location and size of inherent flaws were randomly distributed[9-15]. The energy release rate of each flaw was then calculated using a finite element method and the propagation of the each flaw was evaluated using the Monte Carlo simulation and Eq. (2.2).

For the through-the-thickness flaw models, the new crack was assumed to occur when the rate of energy to crack density is the same as the energy necessary to create new crack surfaces(Eq. 2.3.)

$$G=-\frac{1}{h_{90}}\frac{dU}{dN} \quad \cdots\cdots (2.3)$$

where h_{90} is the thickness of 90° plies, U and N are total energy of the cracked laminate and crack density, respectively. This is the same as the Griffith criterion except using crack density N rather than crack length a. However, three different criteria were proposed depending on the way to interpret right hand side of Eq. (2.3).

The first criterion, which was proposed by the authors, is that the crack density changes from N to N+1 when the energy released in this process reaches a critical value[23-26]. Equation(2.4) shows the way to calculate the energy release rate for this criterion.

$$G=-\frac{1}{h_{90}}(U(N+1)-U(N)) \quad \cdots\cdots (2.4)$$

where U(N) is the total energy of the laminate which has crack density N. Experimentally, a gradual rather than abrupt change in crack density is observed and the crack spacing is not uniform. Thus, the energy change from state i to i+1 in Fig. 5-a was calculated from Fig. 5-b.

The second criterion is that the crack density doubles from N to 2N when the energy released in this process reaches a critical value(Eq. 2.5) [19, 35].

$$G=-\frac{1}{h_{90}}\frac{(U(2N)-U(N))}{N} \quad \cdots\cdots (2.5)$$

In a cell bounded by one crack at each end, a new crack will form at the middle when the accompanying energy release rate reaches a critical vlaue, Fig. 5-c. Thus, this model can predict the stress required to go from crack density N to 2N. Crack densities between N and 2N are not permitted. Thus, the predicted relation between applied

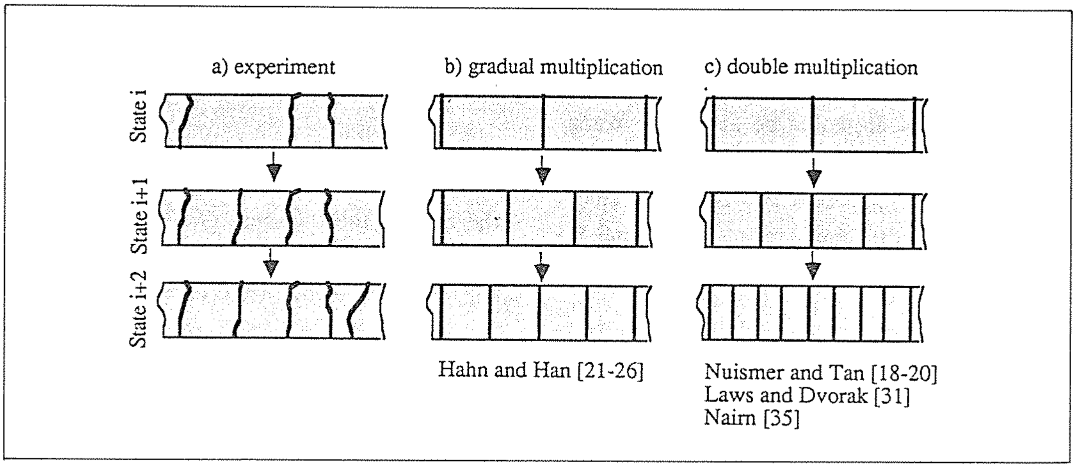

Fig. 5 Schematic diagrams of the ply crack multiplication simulation.

stress and crack density would be stepwise.

The third criterion is that the crack density doubles from N to 2N when the energy release rate calculated from the probability density function reaches a critical value[31]. The probability density function was chosen to be proportional to the stress in the transverse ply between two adjacent ply cracks. The condition of this model to the abrupt change in crack density is the same as the second criterion.

In Fig. 6, the stress vs. crack density predicted using the above criteria are compared with the experimental results for [0/90$_2$]$_s$ T300/934 laminate. For the first criterion, the predicted curve has a relatively long initial plateau indicating that many cracks form almost simultaneously without an increase in load. However, experimental data show more gradual crack multiplication. The initial plateau is as expected since the energy release rate at low crack densities does not change rapidly with applied load. A finite element analysis shows that the perturbed stress in transverse ply reaches its maximum value when its length from the crack surface is about 4 or 5 times the transverse ply thickness[15, 22, 37].

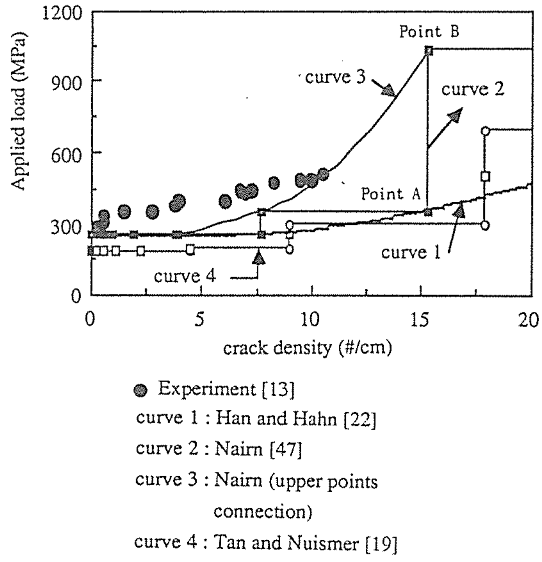

Fig. 6 Applied stress vs. crack density in a T400/934 [0/90$_2$]s laminate.

Thus, if the adjacent crack is further away than the above length, the two cracks will not interact with each other. The authors[23] explained this poor experimental correlation as a result of the fracture toughness being inhomogeneous in composite laminates and proposed a resistance curve to characterize progressive ply cracking. This model was further extended to predict ply crack multiplication under general in-plane loading[26] using Hahn's mixed mode fracture criterion[38].

For the second criterion(Tan and Nuismer[20], Nairn[35]), there will be two stress points(e.g. points A and B in Fig. 6) at each crack density. One is the minimum stress(point A) and the other is the maximum stress(point B) that can be applied at crack density N. Tan and Nuismer noted this aspect and proposed use of average values. Nairn did not note the double multiplication and drew a continuous line(curve 3) connecting maximum stress points only(point B). Nairn obtained a curve steeper than curve 1 and credited the higher slope to a better stress analysis, which he adopted from Hashin[47]. However, his stress analysis is not basically different from those of others. The curve connecting the minimum points(point A) in almost identical to the curve obtained by the first criterion (curve 1). Moreover, the curve still has an initial plateau and shows deviation especially in the lower crack density region. Validity of the second criterion depends on the raionale about 1) the assumption that crack density always jumps twice, 2) the curve merely connecting those few points to make it continuous.

Another aspect is the effect of the out-of-plane stress term. Curve 1 does not include the out-of-plane stresses in its stress analysis, whereas curves 2 and 4 do. Since it turns out all three curves have basically the same trend, it can be conjectured that the effect of the out-of-plane stress term may not be significant to the calculation of energy release rate.

2-3. Stiffness Degradation

Once the stress distribution of the cracked laminate is known, prediction of the stiffness degradation as a function of crack density is quite straightforward. Most models set up the damaged laminate stiffness matrix from the constitutive relations. Some models[46, 47, 49] calculate it from the strain energy. Hashin[47] derived the lower bound solution from the complementary strain energy, while others obtained an upper bound solution from the strain energy. Table 1 listed the stiffnesses that each model can predict.

Fig. 7 shows the general trends of property degradations in graphite/epoxy laminate as a function of transverse ply crack density. The ratio of property degradations drops very fast initially and then approaches the limiting values as the crack density increases. Poisson's ratio and shear modulus experience significant and faster degradation rates when compared to the longitudinal Young's modulus. This is to be expected since the fiber properties dominate the logitudinal modulus while matrix properties are more significant in the Poisson's ratio and shear modulus for this laminate. Fig. 7 also implies that all the elastic constants need to be considered to analyze the behavior of the composite laminate after first ply failure as was stated in Ref. [40] earlier.

Recently, the authors included two interesting features in the calculation of stiffness degradation [26]. One is the effect of longitudinal cracks(Fig. 1). Since ply cracks are observed to appear in both sublaminates, i.e., 0° and 90° plies when a symmetric laminate is under tensile fatigue loading [51], the property degradation will be accelerated to some extent. The equations, in which longitudinal cracks are taken into account, are derived for Poisson's ratio and logitudinal and shear moduli. Thus, the stiffness degreadation of a $[\pm 45]_s$ laminate under tensile loading could be predicted. However, we needed to consider the nonlinearity of the shear property for better experimental correla-

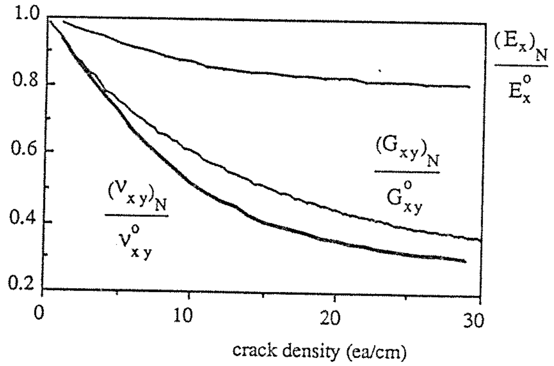

Fig. 7 Mechanical property degradations of graphite/epoxy cross-ply laminate with transverse cracks.

tions. The other is the effect of residual stress. Once ply cracks are formed, permanent deformation will take place due to the residual stresses in the damaged laminates. Thus, the secant modulus of the cracked laminate will be affected. This effect was successfully taken into account in the calculation of longitudinal modulus and Poisson's ratio.

Another aspect is the curved crack, which is most commonly observed when the thickness of the transverse ply is more than 3 plies. Allen et al. have tried to relate the effect of curved cracks to the out of plane stiffness using internal state variables[44, 45]. They also tried to find the reason for the formation of curved cracks using finite element analysis. It is conjectured that the direction of the maximum principal stress close to the straight matrix crack is responsible for the crack path.

For the purpose of comparison, predicted stiffness decreases for $[0/90_3]_s$ glass-epoxy laminate are shown in Table 2. Material properties and experimental results are referred from Ref. [39] and [46]. Most results fall between lower and upper bounds. It is noted the experimental result is even lower than the lower bound solution. Since Aboudi, Allen, Nuismer, and Hashin included out-of-plane stress terms in their analyses, the effect of the out of plane stress on the accuracy of stiffness calculation is not clear.

3. Ply Cracking Under Fatigue Loading

Meanwhile, to explore the crack multiplication under fatigue loading, only one model has been proposed by Chou and Wang[13]. They proposed a model based on the assumption that 1) the growth of a through-the-width type inherent flaw was the mechanism of ply cracking, 2) a power law between the crack growth rate and energy release rate could delineate the formation of ply cracking, 3) new cracks form when an effective flaw grows to a critical length. Parameters were adjusted from a statistical flaw distribution via Monte Carlo simulation. Comprehensive experimental data on ply cracking under static and fatigue loadings could be found in Ref. [13].

In 1989, Boniface and Ogin[52] showed that the fatigue crack growth of transverse ply cracks can be represented by the Paris law. They used transparent glass/epoxy cross-ply laminates for ease of crack detection. Cracks were noted to start not only from the specimen edges but also from the inside of the specimen. They also observed that the crack growth rate depended not on the crack

Table 2. Predictions of stiffness degrdation for $[0/90_3]$s glass-epoxy laminate

	Experiment	Aboudi	Allen	Highsmith	Han	Nuismer	Garrett	Hashin
ΔE_x(%)	45.0	25.0	25.0	29.0	35.5	36.0	37.6	43.7

length but on the crack density. This can be explained by the through-the-thickness flaw concept since the energy release rate of the flaw depends only on crack density which is not true for the through-the-width inherent flaws. The applicability of Paris law means the energy release rate is a critical parameter under fatigue loading.

Recently, the author has found a simple equation to predict the crack density as a function of fatigue cycles[25]. The method is based on the concept of the thorugh-the-thickness inherent flaw, the Paris law, and the resistance curve which has been proposed by the authors. Specifically, the required number of cycles to have crack density N in a laminate can be obtained from Eq.(2.6).

$$n = A \int_0^N \frac{dN}{\left(\frac{\Delta G}{G^R}\right)^{\alpha}} \qquad (2.6)$$

Here n is the required cycles, ΔG and G^R are the range of the energy release rate and the resistance to ply cracking, respectively. A and α are crack multiplication constants in fatigue and are parameters to be determined. With Eq. (2.6), the stress-strain relations can be predicted as functions of applied cycle in fatigue. A design methodology also have been proposed for laminates with ply cracking. The detail of the model is discussed in Ref. [25].

4. Conclusions

A critical review of different models that have been proposed to characterize ply cracking behavior was provided. Assumptions regarding important parameters to ply cracking are delineated to distinguish various models from one another. These models were classified by fracture criterion and the method of stress analysis. Some models were compared to experimental data in first ply cracking, progressive ply cracking, and stiffness degradation. Generally, good correlations were achieved to first ply cracking strain and stiffness degradation. A failure criterion to predict progressive ply cracking by doubling crack density needed more rationalization.

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